



دفتر :

اقتصاد هندسي Engineering Economy

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للطالبة

اللجنة الأكاديمية لقسم الهندسة الصناعية

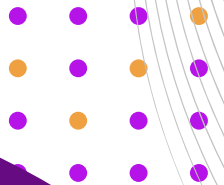
2025



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Engineering Economy

Chapter 1: Introduction to Engineering Economy

The purpose of this book is to develop and illustrate the principles and methodology required to answer the basic economic question of any design: Do its ⁽⁺⁾ benefits exceed its cost? ⁽⁻⁾

خلافات و مبادی

للإجابة على سؤال متعلق بالتصميم

Same meaning

- Design: تصميم
- Solution: حل
- Project: مشروع
- Alternative: بديل

B(+): Advantages: revenues, Sales, Savings.

C(-): Disadvantages: Expenses, Costs.

$\uparrow B \downarrow C \Rightarrow \text{Valt}$
 $(B > C) \Rightarrow \text{Attractive Good Accepted}$

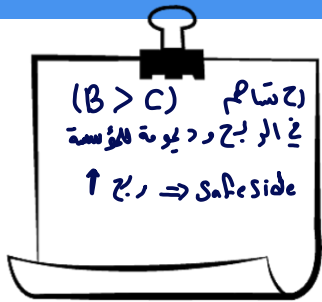
Engineering economy...

مجموعة خطوات لطبق صفة نسون C, B

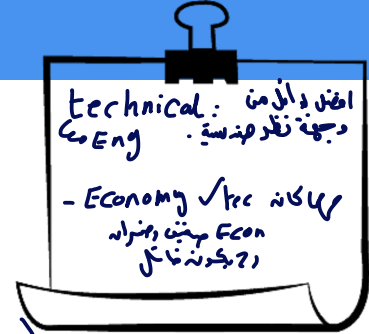
involves the systematic
evaluation of the economic
merits of proposed solutions
to engineering problems.

(فزايا, ايجابيات)

دائما كاد
انه تكون
مزايا واجبايات
أعلى!!



Solutions to engineering problems must (أمثلة على بدائل وحلول)



- promote the well-being and survival of an organization,
- embody creative and innovative technology and ideas,
- permit identification and scrutiny of their estimated outcomes, and
- translate profitability to the “bottom line” through a valid and acceptable measure of merit.

example
engron
الانذار 1.85
الانذار 1.99
cost ↓ ✓ cost ↑ } مثال
ما في توازن بين D + C

« مبدأ في الاقتصاد »

There are **seven fundamental** principles of engineering economy.

Example:

Traffic lights:

Alternative 1: old lights 150W → 10%

Alternative 2: New lights 15W

result:

A₁

A₂

\$ 440K → 10% → \$ 44K

الموتير = التونا بيا السعر 2 (440 - 44) = 396K → 39%K

توزيعات كوكور

Installation Cost → 150K \$

نقطة انكسار

(فيسيا - غير الموزع) 4 > C

- Develop the alternatives (تطوير البدائل)

(المتغير المتكرر المتكرر)

- Focus on the differences ⇒

لازم يكون البدائل مختلفا
Fcost
PBL
الزبد
ما يفرق انما
اختلاف اي دمج
مميز
تقار بالبدائل المتباينين

- Use a consistent viewpoint (وجهة النظر) = وجهة مشتركة

- Use a common unit of measure (\$) (تطابق ال Alternatives)

- Consider all relevant criteria (توضيح المعايير الى ع اساسها في اختيار البدائل الافضل)

(1) بتوزيعية طابور كوكور
(2) خمسة الوكان
(3) Cost J, S

- Make uncertainty explicit (ملاحظة بديل
الواحد المتباين (dis x, n, s))

- Revisit your decisions

Feed Back

خطا 2
1500 كد 1000 كد
مراجعة
استخرج
جديد

Engineering economic analysis procedure

- Problem definition
- Development of alternatives
- Development of prospective outcomes
- Selection of a decision criterion
- Analysis and comparison of alternatives.
- Selection of the preferred alternative.
- Performance monitoring and postevaluation of results.

Example

Oil change problem: (15,000 Km)

Oil Type 1 ✓	Oil Type 2
6 JD/L 5000 Km	4 JD/L 3000 Km
* Filter 5JD	5 JD
* Oil add-on 4L	4L
* Oil change 3 (15000/5000)	5 (15000/3000)

النتيجة:

(1) $6 \times 4 + 5 = 29 \times 3 = 87 \text{ JD/Km}$

(2) $4 \times 4 + 5 = 21 \times 5 = 105 \text{ JD/Km}$

المراد: توفير التكاليف

Electronic spreadsheets are a powerful addition to the analysis arsenal.

- Most engineering economy problems can be formulated and solved using a spreadsheet.
- Large problems can be quickly solved.
- Proper formulation allows key parameters to be changed.
- Graphical output is easily generated.

Engineering Economy

Chapter 2: Cost Concepts and Design Economics

The objective of Chapter 2 is to analyze short-term alternatives when the time value of money is not a factor.

* Alternative "study period" < 1 year
"Useful life"

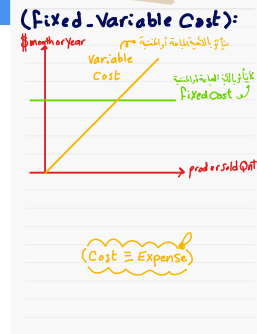
- Time value of money can be ignored (neglected) is not important

- Present economy studies (short-term All)

Examples :

- Choosing the best site to setup portable project equipment
- * transportation cost * rental cost
- production of electronic devices
- Energy efficiency \Rightarrow Profit: saving money quickly!
- airline operations
- Most economic material (Machine sequence, best operating condition for a plant)

Costs can be categorized in several different ways.

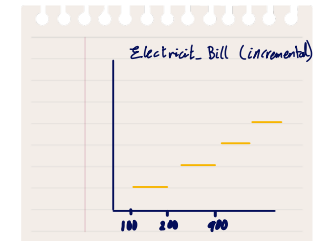
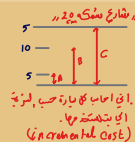


- **Fixed cost:** unaffected by changes in activity level
(تكاليف ثابتة)
- **Variable cost:** vary in total with the quantity of output (or similar measure of activity)
(تكاليف متغيرة)
- **Incremental cost:** additional cost resulting from increasing output of a system by one (or more) units
(Or marginal cost, *Or marginal cost*)
(to make Decision)

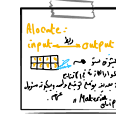
— Produced Quantity
— Sold Quantity
— Production Volume
— Demand Quantity

Example:

A: Small cars
B: medium cars
C: Trucks



More ways to categorize costs



- **Direct:** can be measured and allocated to a specific work activity
- **Indirect:** difficult to attribute or allocate to a specific output or work activity (also *overhead or burden*)
- **Standard cost:** cost per unit of output, established in advance of production or service delivery

• تكاليف المواد والأجور تكون مباشرة (Direct Material, Labor).

• ماخوذة من مكان غير مباشر.
• هي ذات الطبيعة الاندفاعية، والتي لا يمكن عزلها عن المنتج، الأرباح الناتجة عن الإنتاج، (جميعها يربطونهم وبين المنتج)

إذا كان من مذكورة في (S.C) له وجه نظري، أي أنه منتج غير مباشر.

$$\begin{aligned}
 &= \frac{DM + DL + IDC}{Q + Y} \\
 &= \$ / \text{Unit} \\
 &= 5 \$ / \text{Unit}
 \end{aligned}$$

بمجرد معرفة المواد والأجور، يمكن معرفة التكلفة المباشرة.
 حيثما يوجد التكلفة المباشرة، يمكن معرفة التكلفة المباشرة.
 حيثما يوجد التكلفة المباشرة، يمكن معرفة التكلفة المباشرة.

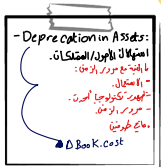
- Selling Price : \$ / Unit
 لا يمكن أن يكون أعلى من (S.C)
 - Actual cost :
 كما قال في السابق، يتم حساب التكاليف الفعلية.
 مثال: إذا كانت التكلفة الفعلية (Selling Price) أعلى من التكلفة المباشرة (Actual Cost)، فهذا يعني أن التكلفة المباشرة (Actual Cost) هي التي يجب استخدامها.

Some useful cost terminology

- **Cash cost:** a cost that involves a payment of cash.



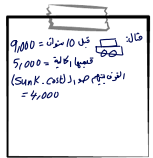
- **Book cost:** a cost that does not involve a cash transaction but is reflected in the accounting system.



(depreciation in Asset)
محصلة حسابات المحاسبة

	Market value (القيمة السوقية)	Book Value (القيمة المحاسبية)
t=0	5,000	5,000
t=1	4,500	4,000
t=2	3,000	2,000
t=6	0	0
t=7	-100	0

- **Sunk cost:** a cost that has occurred in the past and has no relevance to estimates of future costs and revenues related to an alternative course of action.



بمستوى المحاسبة

Ex: عند تقييم خيار أو البديل الاستثماري، لا تأخذ في الاعتبار التكاليف التي تم إنفاقها في الماضي، بل تأخذ في الاعتبار التكاليف المستقبلية.

More useful cost terminology

- تكلفة الفرصة البديلة (الغائبة)
- الفرصة التي كان يمكن أن نحققها لو اخترنا خياراً آخر بدلاً من الخيار الذي اخترناه. (أي أننا اخترنا خياراً معيناً فليس لدينا خيار آخر)
- **Opportunity cost**: the monetary advantage foregone due to **limited resources**. The cost of the best rejected opportunity.
- | | Capital Investment | Profit |
|------------|--------------------|--------|
| Best A_1 | 14,000 | 7,000 |
| Lost A_2 | 12,000 | 5,000 |
| A_3 | 10,000 | 2,000 |
- البدائل الثلاثة معقبة
- لو كان لدينا خيار آخر (أي أننا اخترنا خياراً معيناً فليس لدينا خيار آخر)
- الفرصة التي كان يمكن أن نحققها لو اخترنا خياراً آخر بدلاً من الخيار الذي اخترناه. (أي أننا اخترنا خياراً معيناً فليس لدينا خيار آخر)
- **Life-cycle cost**: the summation of all costs related to a product, structure, system, or service during its life span.

$$\text{Life cycle cost} = \text{Initial cost} + \text{Operation and Maintenance} + \text{Disposal cost}$$

(بداية المشروع) $(t=1 \rightarrow n)$ (خاتمة المشروع) $(t=n)$

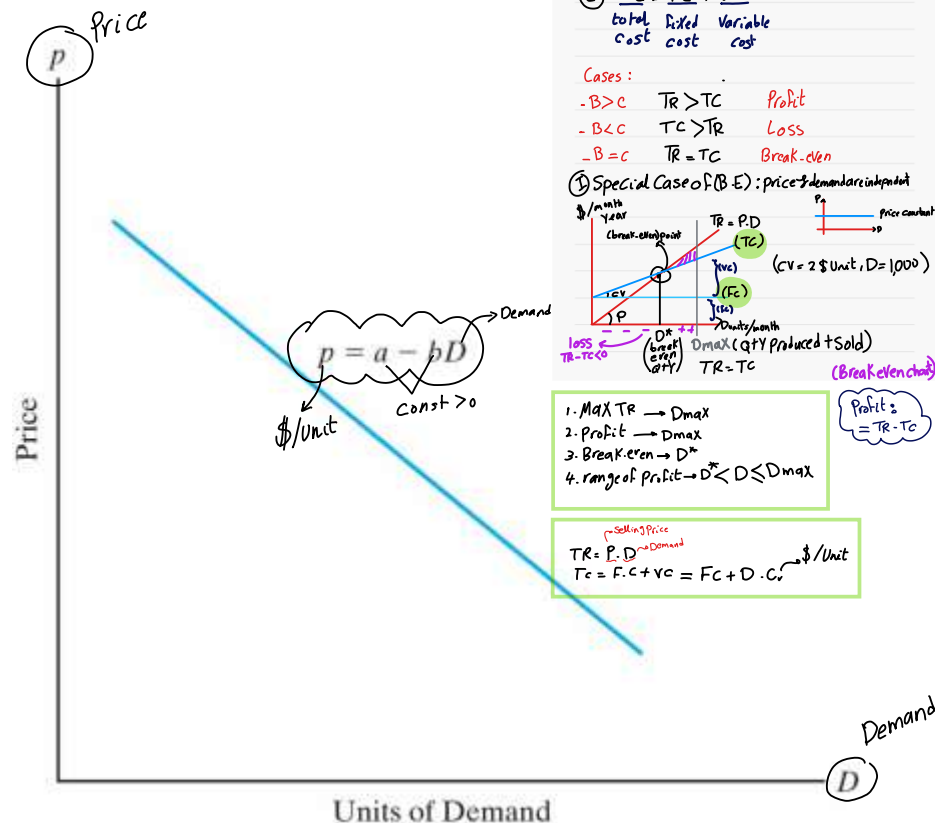
المجموع الكلي للتكاليف خلال مدة المشروع

The general price-demand relationship

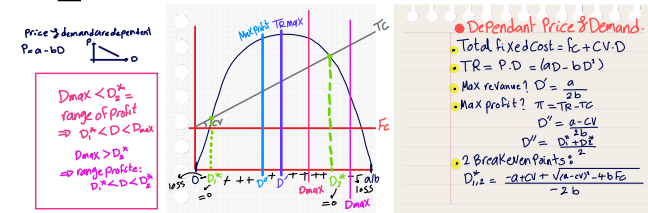
Break-even Cases:

- 1) ind. ($P \nmid D$)
 $P = \text{constant}$
- 2) dep. ($P \nmid D$)
 $P = a - bD$

The demand for a product or service is directly related to its price according to $p = a - bD$ where p is price, D is demand, and a and b are constants that depend on the particular product or service.



Total revenue depends on price and demand.



Total revenue is the product of the selling price per unit, p , and the number of units sold, D .

$$TR = pD = (a - bD)D = aD - bD^2$$

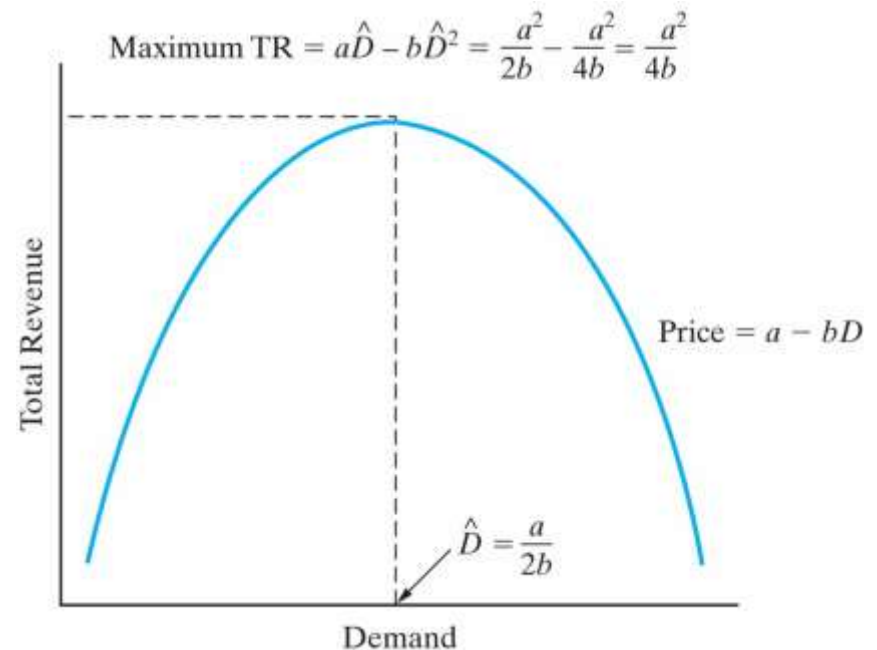
$$\text{for } 0 \leq D \leq \frac{a}{b} \quad \text{and} \quad a > 0, \quad b > 0$$

Calculus can help determine the demand that maximizes revenue.

$$\frac{dTR}{dD} = a - 2bD = 0$$

Solving, the optimal demand is

$$\hat{D} = \frac{a}{2b}$$



We can also find maximum profit...

Profit is revenue minus cost, so

$$\text{Profit} = -bD^2 + (a - c_v)D - C_F$$

for

$$0 \leq D \leq \frac{a}{b} \quad \text{and} \quad a > 0, b > 0$$

Differentiating, we can find the value of D that maximizes profit.

$$D^* = \frac{a - c_v}{2b}$$

And we can find revenue/cost breakeven.

Breakeven is found when total revenue = total cost.
Solving, we find the demand at which this occurs.

$$D' = \frac{-(a - c_v) \pm \sqrt{(a - c_v)^2 - 4(-b)(-C_F)}}{2(-b)}$$

Engineers must consider cost in the design of products, processes and services.

- “Cost-driven design optimization” is critical in today’s competitive business environment.
- In our brief examination we examine discrete and continuous problems that consider a single primary cost driver.

Two main tasks are involved in cost-driven design optimization.

1. Determine the optimal value for a certain alternative's design variable.
2. Select the best alternative, each with its own unique value for the design variable.

Cost models are developed around the design variable, X .

Optimizing a design with respect to cost is a four-step process.

- Identify the design variable that is the primary cost driver.
- Express the cost model in terms of the design variable.
- For continuous cost functions, differentiate to find the optimal value. For discrete functions, calculate cost over a range of values of the design variable.
- Solve the equation in step 3 for a continuous function. For discrete, the optimum value has the minimum cost value found in step 3.

Here is a simplified cost function.

$$\text{Cost} = aX + \frac{b}{X} + k$$

where,

a is a parameter that represents the directly varying cost(s),

b is a parameter that represents the indirectly varying cost(s),

k is a parameter that represents the fixed cost(s), and

X represents the design variable in question.

“Present economy studies” can ignore the time value of money.

- Alternatives are being compared over one year or less.
- When revenues and other economic benefits vary among alternatives, choose the alternative that maximizes overall profitability of defect-free output.
- When revenues and other economic benefits are not present or are constant among alternatives, choose the alternative that minimizes total cost per defect-free unit.

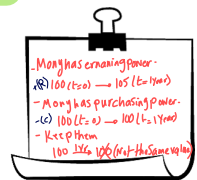
Engineering Economy

Chapter 4: The Time Value of Money

The objective of Chapter 4 is to explain time value of money calculations and to illustrate economic equivalence.

Money has a time value.

- ^{(t=0) below} **Capital** refers to wealth in the form of money or property that can be used to produce more wealth.
- ^{long-term} **Engineering economy studies** involve the commitment of capital for extended periods of time.
- A dollar today is worth more than a dollar one or more years from now (for several reasons).



Return to capital in the form of interest and profit is an essential ingredient of engineering economy studies.

- Interest and profit pay the providers of capital for forgoing its use during the time the capital is being used.
- Interest and profit are payments for the risk the investor takes in letting another use his or her capital.
- opportunity cost
- Any project or venture must provide a sufficient return to be financially attractive to the suppliers of money or property.

Simple Interest: infrequently used

When the total interest earned or charged is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods, the interest and interest rate are said to be *simple*.

Principal
(المبلغ الأصلي)

معدل الفائدة

عدد
السنوات

Computation of simple interest

فائدة بسيطة

The total interest, \underline{I} , earned or paid may be computed using the formula below.

$$\underline{I} = (P)(N)(i)$$

Diagram annotations:
 - \underline{I} : total intrst
 - P : Principle amount
 - N : Period
 - i : interest rate

P = principal amount lent or borrowed

المبلغ الذي قُدم لها

N = number of interest periods (e.g., years)

عدد الفترات الزمنية

i = interest rate per interest period

(سعر الفائدة)

$$F = P + I$$

Diagram annotations:
 - F : future amount
 - P : Present
 - I : interest
 - i : interest rate

The total amount repaid at the end of N interest periods is $P + \underline{I}$.

total accumulated
 Total owed amount after (n) years
 Future amount

If \$5,000 were loaned for five years at a simple interest rate of 7% per year, the interest earned would be

$$\underline{I} = \$5,000 \times 5 \times 0.07 = \$1,750$$

So, the total amount repaid at the end of five years would be the original amount (\$5,000) plus the interest (\$1,750), or \$6,750.

{
- earned
- charged
- owed

Compound interest reflects both the remaining principal and any accumulated interest. For **\$1,000 at 10%** (Year)
 كل سنة بنسبة 10% فائدة

Period	(1) Amount owed at beginning of period	(2)=(1)x10% Interest amount for period	(3)=(1)+(2) Amount owed at end of period
1	\$1,000	\$100	\$1,100
2	\$1,100	\$110	\$1,210
3	\$1,210	\$121	\$1,331

Compound interest is commonly used in personal and professional financial transactions.

$$F = P(1+i)^N$$

Economic equivalence allows us to compare alternatives on a common basis.

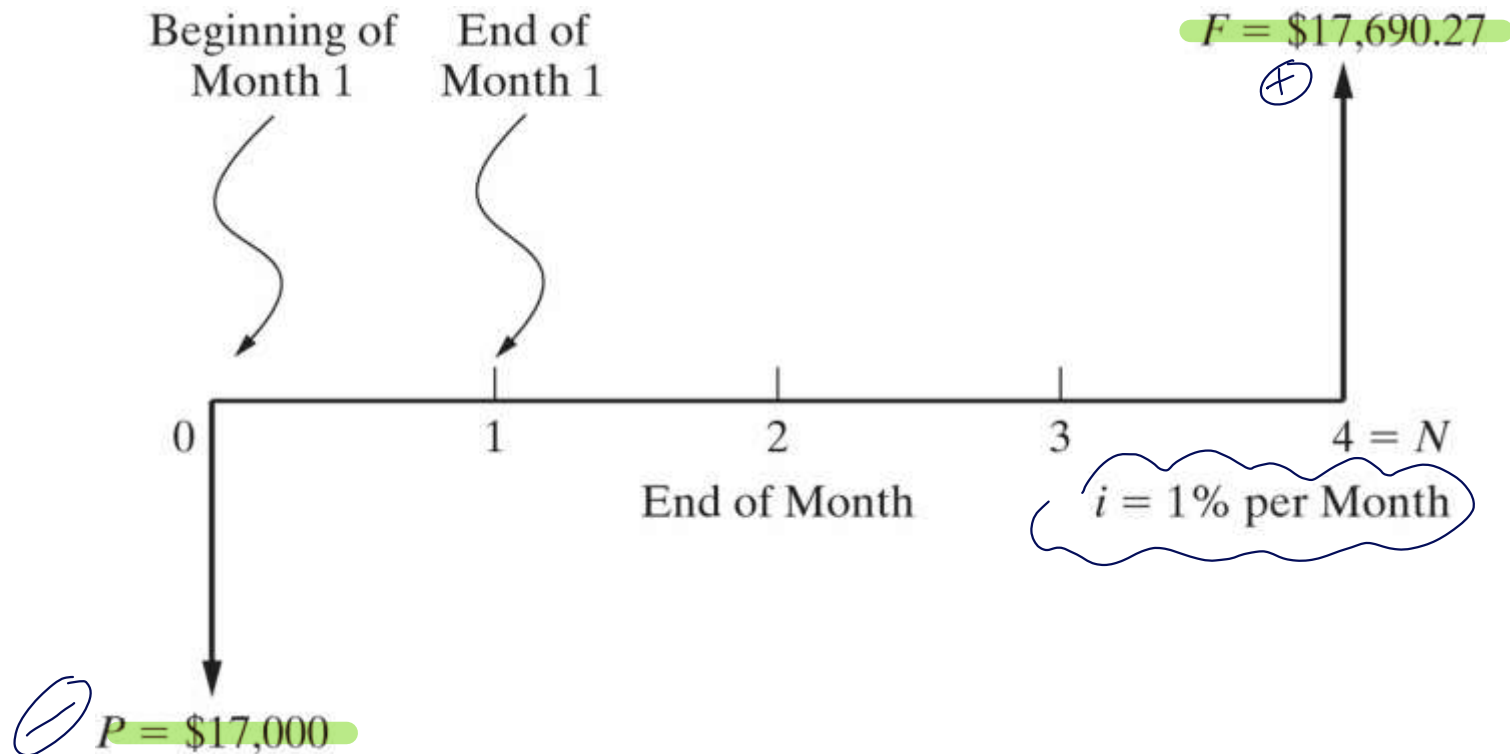
- Each alternative can be reduced to an *equivalent basis* dependent on
 - interest rate,
 - amount of money involved, and
 - timing of monetary receipts or expenses.
- Using these elements we can “move” cash flows so that we can compare them at particular points in time.

- Cash-flow:
1. Cash amount (\$)
 2. Timing ($t=0, t=n$)
 3. interest rate (i)
 4. View point
 - lender up fees
 - Borrowed up fees

We need some tools to find economic equivalence.

- Notation used in formulas for compound interest calculations.
 - i = effective interest rate per interest period
 - N = number of compounding (interest) periods
 - P = present sum of money; *equivalent* value of one or more cash flows at a reference point in time; the present
 - F = future sum of money; *equivalent* value of one or more cash flows at a reference point in time; the future
 - A = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

A cash flow diagram is an indispensable tool for clarifying and visualizing a series of cash flows.



Cash flow tables are essential to modeling engineering economy problems in a spreadsheet

			$= -25000 - 9400$	$= C3 - B3$	$= \text{SUM}(D\$3:D3)$
	A	B	C	D	E
1		Alternative A	Alternative B	Difference	Cumulative
2	End of Year	Net Cash Flow	Net Cash Flow	(B-A)	Difference
3	0 (now)	\$ (18,000)	\$ (60,000)	\$ (42,000)	\$ (42,000)
4	1	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (32,600)
5	2	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (23,200)
6	3	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (13,800)
7	4	\$ (34,400)	\$ (34,400)	\$ -	\$ (13,800)
8	5	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (4,400)
9	6	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ 5,000
10	7	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ 14,400
11	8	\$ (32,400)	\$ (17,000)	\$ 15,400	\$ 29,800
12	Total	\$ (291,200)	\$ (261,400)		

$= -34400 + 2000$
 $= \text{SUM}(B3:B11)$
 $= -25000 + 8000$

We can apply compound interest formulas to “move” cash flows along the cash flow diagram.

Using the standard notation, we find that a present amount, P , can grow into a future amount, F , in N time periods at interest rate i according to the formula below.

$$F = P(1 + i)^N$$

In a similar way we can find P given F by

$$P = F(1 + i)^{-N}$$

It is common to use standard notation for interest factors.

$$(1 + i)^N = (F/P, i, N)$$

This is also known as the *single payment compound amount* factor. The term on the right is read “ F given P at $i\%$ interest per period for N interest periods.”

$$(1 + i)^{-N} = (P/F, i, N)$$

is called the *single payment present worth* factor.

We can use these to find economically equivalent values at different points in time.

\$2,500 at time zero is equivalent to how much after six years if the interest rate is 8% per year?

$$F = \$2,500(F/P, 8\%, 6) = \$2,500(1.5869) = \$3,967$$

\$3,000 at the end of year seven is equivalent to how much today (time zero) if the interest rate is 6% per year?

$$P = \$3,000(P/F, 6\%, 7) = \$3,000(0.6651) = \$1,995$$

There are interest factors for a series of end-of-period cash flows.

$$F = A \left[\frac{(1 + i)^N - 1}{i} \right] = A(F/A, i\%, N)$$

How much will you have in 40 years if you save \$3,000 each year and your account earns 8% interest each year?

$$F = \$3,000(F/A, 8\%, 40) = \$3,000(259.0565) = \$777,170$$

Finding the present amount from a series of end-of-period cash flows.

$$P = A \left[\frac{(1 + i)^N - 1}{i(1 + i)^N} \right] = A(P/A, i\%, N)$$

How much would is needed today to provide an annual amount of \$50,000 each year for 20 years, at 9% interest each year?

$$P = \$50,000(P/A, 9\%, N) = \$50,000(9.1285) = \$456,427$$

Finding A when given F.

$$A = F \left[\frac{i}{(1+i)^N - 1} \right] = F(A/F, i\%, N)$$

How much would you need to set aside each year for 25 years, at 10% interest, to have accumulated \$1,000,000 at the end of the 25 years?

$$A = \$1,000,000(A/F, 10\%, 25) = \$1,000,000(0.0102) = \$10,200$$

Finding A when given P.

$$A = P \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right] = P(A/P, i\%, N)$$

If you had \$500,000 today in an account earning 10% each year, how much could you withdraw each year for 25 years?

$$A = \$500,000(A/P, 10\%, 25) = \$500,000(0.1102) = \$55,100$$

It can be challenging to solve for N or i .

- We may know P , A , and i and want to find N .
- We may know P , A , and N and want to find i .
- These problems present special challenges that are best handled on a spreadsheet.

Finding N

Acme borrowed \$100,000 from a local bank, which charges them an interest rate of 7% per year. If Acme pays the bank \$8,000 per year, now many years will it take to pay off the loan?

$$\$100,000 = \$8,000(P/A, 7\%, N)$$

So,

$$(P/A, 7\%, N) = \frac{\$100,000}{\$8,000} = 12.5 = \frac{(1.07)^N - 1}{0.07(1.07)^N}$$

This can be solved by using the interest tables and interpolation, but we generally resort to a computer solution.

Finding i

Jill invested \$1,000 each year for five years in a local company and sold her interest after five years for \$8,000. What annual rate of return did Jill earn?

$$\$8,000 = \$1,000(F/A, i\%, 5)$$

So,

$$(F/A, i\%, 5) = \frac{\$8,000}{\$1,000} = 8 = \frac{(1 + i)^5 - 1}{i}$$

Again, this can be solved using the interest tables and interpolation, but we generally resort to a computer solution.

There are specific spreadsheet functions to find N and i .

The Excel function used to solve for N is

$\text{NPER}(\text{rate}, \text{pmt}, \text{pv})$, which will compute the number of payments of magnitude pmt required to pay off a present amount (pv) at a fixed interest rate (rate).

One Excel function used to solve for i is

$\text{RATE}(\text{nper}, \text{pmt}, \text{pv}, \text{fv})$, which returns a fixed interest rate for an annuity of pmt that lasts for nper periods to either its present value (pv) or future value (fv).

We need to be able to handle cash flows that do not occur until some time in the future.

- Deferred annuities are uniform series that do not begin until some time in the future.
- If the annuity is deferred J periods then the first payment (cash flow) begins at the end of period $J+1$.

Finding the value at time 0 of a deferred annuity is a two-step process.

1. Use $(P/A, i\%, N-J)$ find the value of the deferred annuity at the end of period J (where there are $N-J$ cash flows in the annuity).
2. Use $(P/F, i\%, J)$ to find the value of the deferred annuity at time zero.

$$P_0 = A(P/A, i\%, N - J)(P/F, i\%, J)$$

Sometimes cash flows change by a constant amount each period.

We can model these situations as a *uniform gradient* of cash flows. The table below shows such a gradient.

End of Period	Cash Flows
1	0
2	G
3	$2G$
:	:
N	$(N-1)G$

It is easy to find the present value of a uniform gradient series.

Similar to the other types of cash flows, there is a formula (albeit quite complicated) we can use to find the present value, and a set of factors developed for interest tables.

$$(P/G, i\%, N) = \frac{1}{i} \left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N} \right]$$

We can also find A or F equivalent to a uniform gradient series.

$$(A/G, i\%, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$$

$$(F/G, i\%, N) = \frac{1}{i} (F/A, i\%, N) - \frac{N}{i}$$

The annual equivalent of this series of cash flows can be found by considering an annuity portion of the cash flows and a gradient portion.

End of Year	Cash Flows (\$)
1	2,000
2	3,000
3	4,000
4	5,000

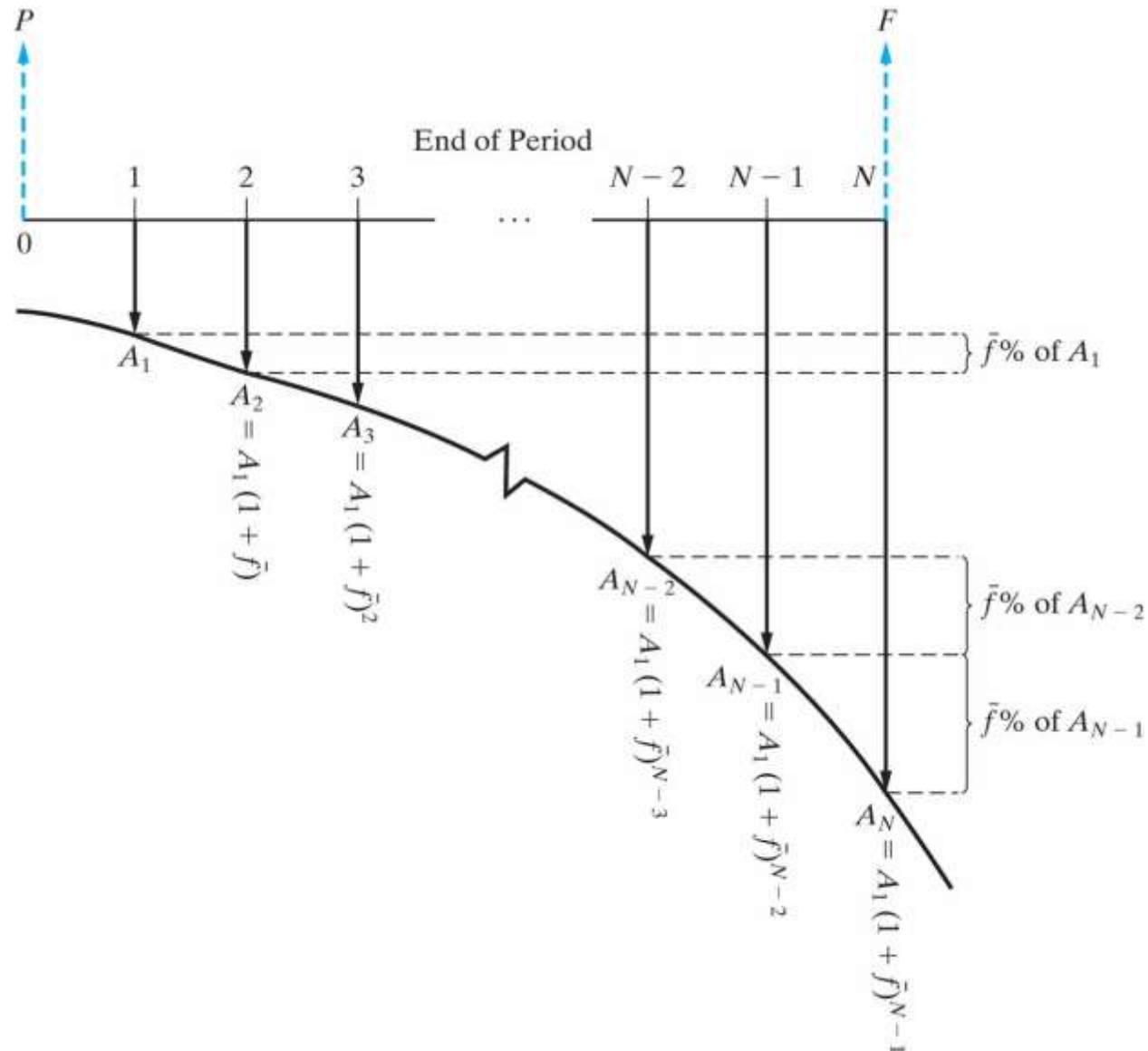
End of Year	Annuity (\$)	Gradient (\$)
1	2,000	0
2	2,000	1,000
3	2,000	2,000
4	2,000	3,000

$$A = \$2,000 + \$1,000(A/G, 8\%, 4) = \$3,404$$

Sometimes cash flows change by a constant rate, \bar{f} , each period--this is a *geometric gradient series*.

This table presents a geometric gradient series. It begins at the end of year 1 and has a rate of growth, \bar{f} , of 20%.

End of Year	Cash Flows (\$)
1	1,000
2	1,200
3	1,440
4	1,728



We can find the present value of a geometric series by using the appropriate formula below.

If $\bar{f} \neq i$

$$\frac{A_1 [1 - (P/F, i\%, N)(F/P, \bar{f}\%, N)]}{1 - \bar{f}}$$

If $\bar{f} = i$

$$A_1 N (P/F, i\%, 1)$$

Where A_1 is the initial cash flow in the series.

When interest rates vary with time different procedures are necessary.

- Interest rates often change with time (e.g., a variable rate mortgage).
- We often must resort to moving cash flows one period at a time, reflecting the interest rate for that single period.

The present equivalent of a cash flow occurring at the end of period N can be computed with the equation below, where i_k is the interest rate for the k^{th} period.

$$P = \frac{F_N}{\prod_{k=1}^N (1 + i_k)}$$

If $F_4 = \$2,500$ and $i_1=8\%$, $i_2=10\%$, and $i_3=11\%$, then

$$P = \$2,500(P/F, 8\%, 1)(P/F, 10\%, 1)(P/F, 11\%, 1)$$

$$P = \$2,500(0.9259)(0.9091)(0.9009) = \$1,896$$

Nominal and effective interest rates.

- More often than not, the time between successive compounding, or the interest period, is less than one year (e.g., daily, monthly, quarterly).
- The annual rate is known as a *nominal* rate.
- A *nominal* rate of 12%, compounded monthly, means an interest of 1% ($12\%/12$) would accrue each month, and the annual rate would be *effectively* somewhat greater than 12%.
- The more frequent the compounding the greater the *effective* interest.

The effect of more frequent compounding can be easily determined.

Let r be the nominal, annual interest rate and M the number of compounding periods per year. We can find, i , the effective interest by using the formula below.

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

Finding effective interest rates.

For an 18% nominal rate, compounded quarterly, the effective interest is.

$$i = \left(1 + \frac{0.18}{4}\right)^4 - 1 = 19.25\%$$

For a 7% nominal rate, compounded monthly, the effective interest is.

$$i = \left(1 + \frac{0.07}{12}\right)^{12} - 1 = 7.23\%$$

Interest can be compounded continuously.

- Interest is typically compounded at the end of discrete periods.
- In most companies cash is always flowing, and should be immediately put to use.
- We can allow compounding to occur continuously throughout the period.
- The effect of this compared to discrete compounding is small in most cases.

We can use the effective interest formula to derive the interest factors.

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

As the number of compounding periods gets larger (M gets larger), we find that

$$i = e^r - 1$$

Continuous compounding interest factors.

$$(P/F, \underline{r}\%, N) = e^{-rN}$$

$$(F/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^r - 1}$$

$$(P/A, \underline{r}\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^r - 1)}$$

The other factors can be found from these.

CHS: Evaluation of Eng.

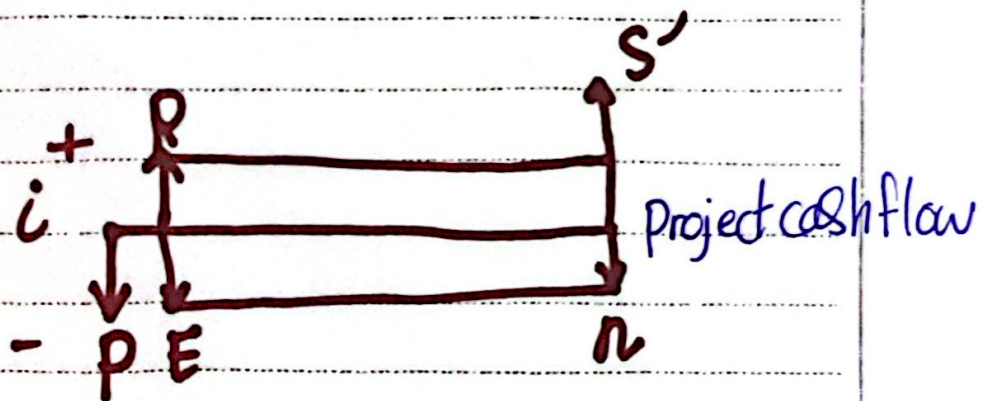
long term alternative

$B > C$ ✓

$C > B$ X

TVM important > 1 year.

• Cash flow:



P : Capital investment / initial investment
($t=0$)

(\$) المبلغ المبدئي وقيمة المبلغ في بداية المشروع. (-)

E : Operation and maintenance Cost
($t=1 \sim n$)

مصاريف وصيانة
(\$/year)

R : revenues, sales, savings (+)
($t=1 \sim n$)

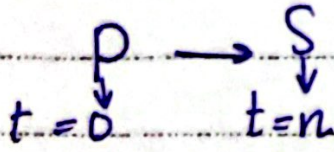
إيرادات (\$/year)

S' : Salvage Value (market value)
($t=n$)

مبلغ بيعه وقت الانتهاء للمشروع
(+/-)

10000 سترينجا به 5 سنين لبيعها 5000 (+)
لما نبيرونا ادفع عليها اكلها طر (-)

الأصول و Assets Value



«فقدت السيارة مع الزمن وقتها»

$$\# \text{ loss in Assets} = (P - S)$$

n alternative / project life - (Useful life)
(study period)

فترة دراسة المشروع

i is minimum attractive rate of return.

نسبة مئوية يجب أن عني وليست ضريبة أقيم المشروع

depends on - source of capital

- Type of project
- Risk
- opportunity cost.

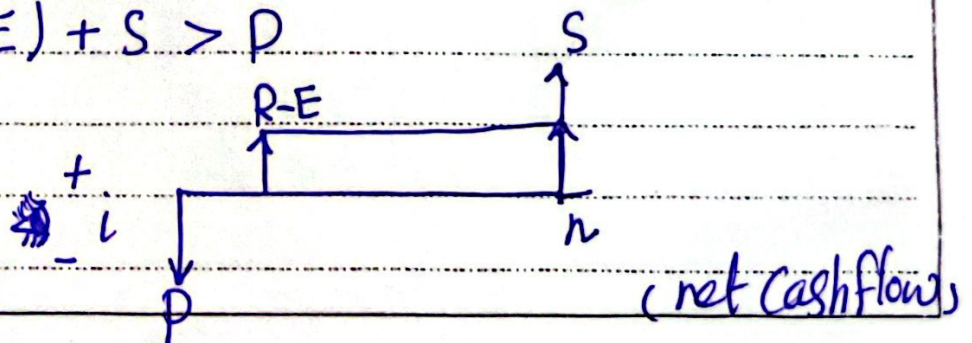
} i: MARR
given

بكل سؤال ربح الرسم ال cashflow أضف بهي أقيم على فترات
أنه (B > C) مقبولون accepted

$$R + S > P + E$$

$$(R - E) > (P - S)$$

$$(R - E) + S > P$$



• Methods used in evaluations

(1) present worth (PW) \$ (t=0)

(2) future worth (FW) \$ (t=n)

(3) Annual worth (AW) \$/Year (t=1~n)

الهادف
في كمية
الربح

• Rate of return

(1) IRR (% per year)

(2) ERR

الهادف
في نسبة الربح

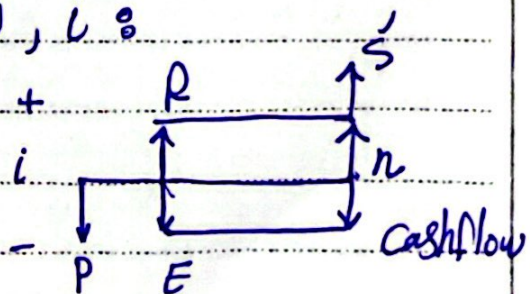
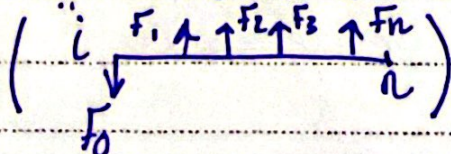
• Pay Back (#(years)) الزمن اللازم لاسترداد رأس المال: الهادف
في الزمن اللازم
حتى اربح.

- Equilant Worth methods:

1- given a project cash flow, i :

(A) net cash flow

- برز عند كل نقطة في النقاط حسب المحلة



لا اذا كان عندى سهمين باقية الفرق بينهم والمباها

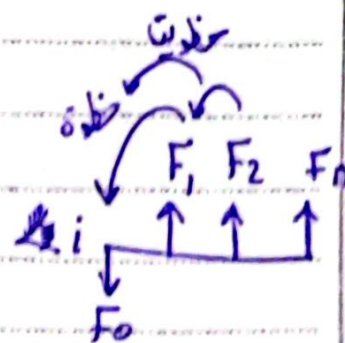
(flow of F)

مع الاكبر

(B) calculation for project

$\Rightarrow PW(i) =$ برجع كل القم عند الصفر

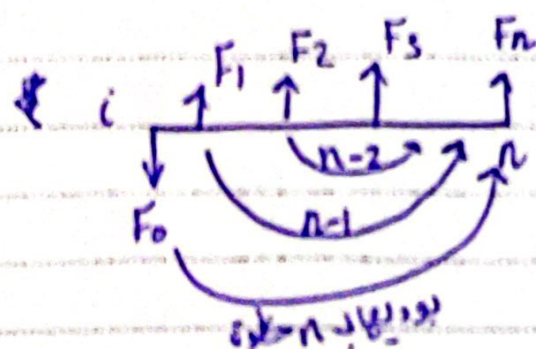
$$\sum_{t=0}^n F_t \cdot (P/F, i, t)$$



فئة F_1 برجعها خطوة، F_2 خطوتين، F_3 ثلاث خطوات، وهكذا

$\Rightarrow FW(i) =$ بودي كل القم عند n

$$\sum_{t=0}^n F_t \cdot (P/F, i, n-t)$$



$\left\{ \begin{array}{l} PW \\ FW \\ AW \end{array} \right\}$ وحدة عمل
لنحسبها
لأهم بديلاً

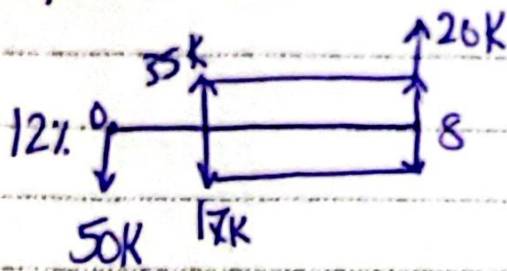
$\Rightarrow AW(i) =$ القم جوليها ل

$$= P(W)(i) \cdot (A/P, i, n)$$

$$\text{or} = FW(i) \cdot (A/F, i, n)$$

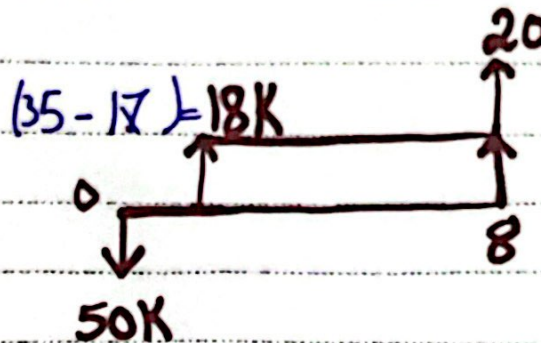
(C) if $PW(i) \geq 0 \rightarrow$ accepted if < 0 not accepted
if $FW(i) / AW(i) \geq 0 \rightarrow$

Example 1 (Slide #4)



Result
(Project is Accepted)

• A / net cash flow



at $n=8$
(20+18) = 38K
(38K)
Cash flow at $n=8$

$$\rightarrow F_0 = -50K \quad | \quad F_1 = 18K \quad | \quad F_8 = 18K + 20K = 38K$$

لا مفعلة 8 سنوات $n=0$ مع اس (ن) باقيا

$$\begin{aligned} \bullet B / \quad PW(12\%) &= -50,000 + 18,000(P/A, 12\%, 8) \\ &\quad + 20,000(P/F, 12\%, 8) \\ &= 47,495 > 0, \text{ Accepted} \end{aligned}$$

$$\begin{aligned} FW(12\%) &= -50,000(F/P, 12\%, 8) + \\ &\quad 18,000(F/A, 12\%, 8) + 20,000 = 117,596\$ \end{aligned}$$

$$AW(12\%) = -50,000(P/A, 12\%, 8) + 18,000 +$$

$$20,000(F/F, 12\%, 8) = 9,560/\text{year}$$

Same example But $i=0\%$.

بستوفيا مثلاً مالي وبتوفيا مثلاً مالي مقبولة 8 سنين

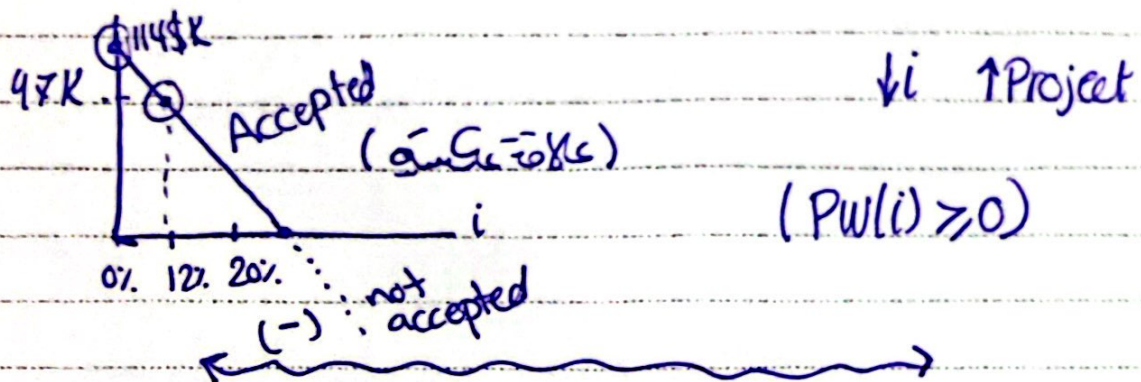
$$PW(0\%) = 18,000 \times 8 + -50,000 + 20,000$$

$$= 114,000 \$$$

$$FW(0\%) = 114,000 \$$$

زى مالي مقبولة 8 سنين ← بوزعها على 8 سنين

$$AW(0\%) = -50,000/8 + 20,000/8 + 18,000 = 14,250 \$ / year$$



Pay back period :

* الزمن اللازم لاسترداد رأس المال المدفوع في بداية المشروع
 - ليست مقياساً لربحية المشروع ولتعتبر مقياساً
 (السيولة (liquidity) •)
 (الربحية ← profitability)

Example : توليد الطاقة الكهربائية من الألواح الشمسية

$$\ominus P = 4,000,000 JD$$

$$\oplus \text{ Saving : } 1,000,000 JD / year$$

الزمن اللازم هو 4 years

• Simple pay Back & "without interest"

$$\text{Min } \theta' : \sum_{t=0}^{\theta'} F_t \geq 0, \quad 0 < \theta' \leq n$$

↓
net cash flow

(1) evaluate net cash flow

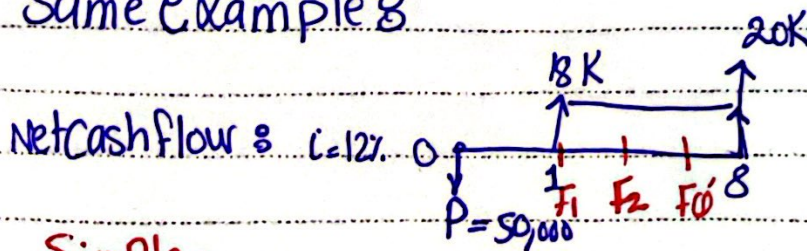
(2) حساب المتبقيات

• Discounted pay Back period & (i)

Min θ''

$$\sum_{t=0}^{\theta''} F_t \cdot (P/F, i, t) \geq 0, \quad 0 < \theta'' \leq n$$

Same Example &



• Simple:

Start at $t=1$: $-50,000 + 18,000 = -32,000$

$t=2$: $-50,000 + 36,000 = -14,000$

$t=3$: $-50,000 + 54,000 = 4,000$

$(t=3) \Rightarrow (\theta' = 3)$

لنسترجع
رأس المال

القيمة الحالية
Discount Value

القيمة الحالية
لنسترجع رأس المال

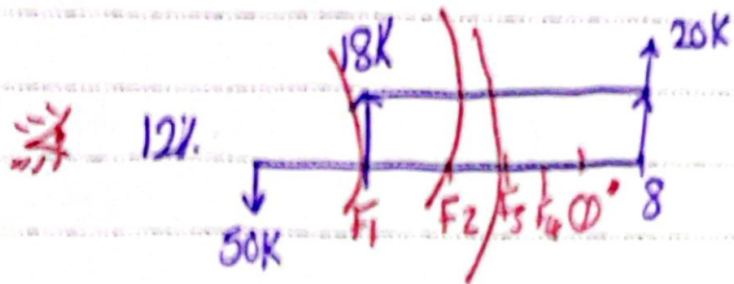
أي برصبي رأس المال
رأس المال
يحتاج إلى اهداقل عدد من السنوات

• يفضل انه تكون ال (PBP) قيمتها كبيرة / صغيرة
 له لا بها حفاضا للمسؤول خلال المشروع

• Discounted:

منه

$$\sum F_t / (1 + i)^t$$



$$F_1 = 18,000 (P/F, 12\%, 1) - 50,000 = -33,000 \times$$

$$F_2 = -50,000 + 18K (P/F, 12\%, 2) = -19,579 \times$$

$$F_3 = -50,000 + 18K (P/F, 12\%, 3) = -6,767 \times$$

$$F_4 = -6,767 + 18K (P/F, 12\%, 4) = 4,672 \checkmark$$

$$(N=4)$$



Mo Tu We Th Fr Sa Su

Memo No. _____

Date / /

• Rate of return methods

(1) internal rate of return : العائد الداخلي للمشروع

for a given project cash flow, MARR :

(A) find the net cash flow

(B) find i^* : $PW(i^*) = 0$

(C) if i^* → Single Value : $IRR = i^*$

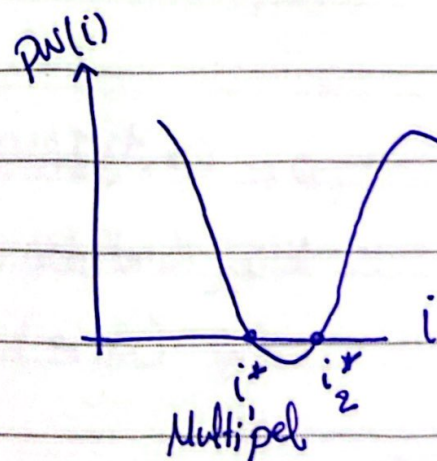
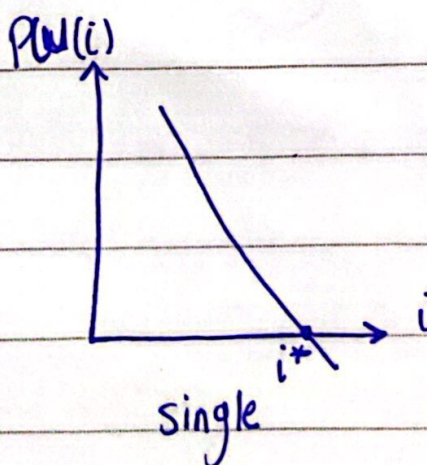
↳ if $IRR \geq MARR$

then Project Accepted

↳ else $< MARR$ X

↳ Multiple Value :

- method fails / use a different method





Mo Tu We Th Fr Sa Su

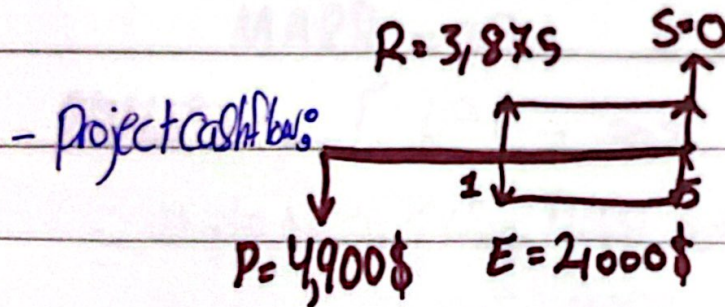
Memo No. _____

Date / /

Example: project has initial investment 4900

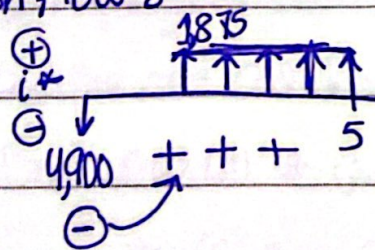
$n=5$, $R=3875\$$, $E=2000\$$

$S=0$, $MARR=25\%$ per year



Answer

(1) net cash flow



$$PW(i^*) \Rightarrow -4900 + 1875 (P/A, i^*, 5) = 0$$

إذا تغير cash flow من (+) إلى (-) أو العكس

الفا (single) و root(1)

$$i^* = 26.5\% \rightarrow IRR = 26.5\% > MARR$$

. Accepted project.

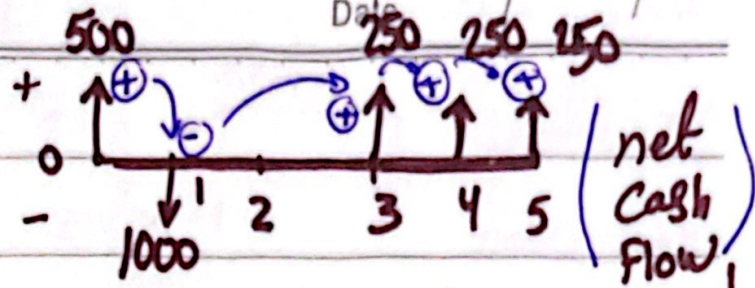


Mo Tu We Th Fr Sa Su

Memo No. _____

Date _____

Example (2) :



لأنه عند كل نقطة في الزمان يكون بالفرق بالقيمة .

MARR = 35%.

$PW(i^*)$: — [$i^*_1 = 30\%$, $i^*_2 = 62\%$]

ما يتوزن في هاتين النقطتين لا يتجاوز
 $\begin{cases} i^*_1 < MARR \text{ not accepted} \\ i^*_2 > MARR \text{ accepted} \end{cases}$

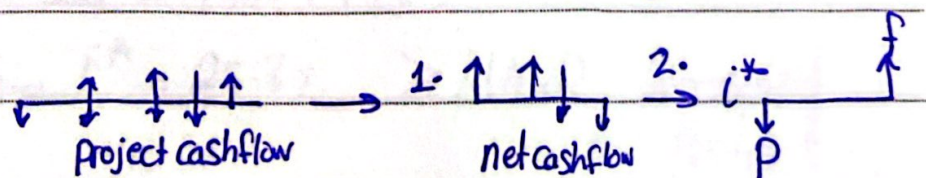
← External Rate of return method (ERR) :

for a given project cash flow, $E = MARR$:

1. Find the net cash flow.

2. Find $PW(E)$ # أي سن (إيجابي) - جيبه عند الصفر

• $FW(E)$ # أي سن لفرق (+) بفرق عند الصفر
 (اعتدع عند N)



3. find $i^* \Rightarrow F = P(1+i^*)^N$ (one root only)

4. $ERR = i^* \rightarrow$ if $ERR > MARR$ accepted

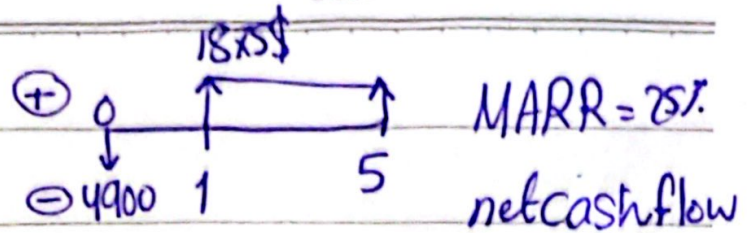
$ERR < MARR$ not accepted



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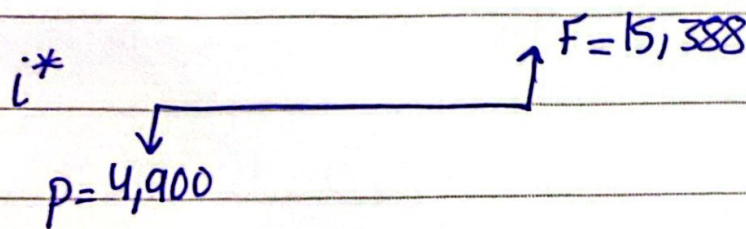


Answer:

$$PW(25\%) = 4900$$

سلف

$$FW_R = 1875(F/A, 25, 5) = 15,388$$



$$i^* = F = P(1+i^*)^n$$

$$15,388 = 4900 (1+i^*)^5$$

سنة العائد $i^* = 25.7\% > MARR$ Accepted Project.